# An Introduction to Partisan Gerrymandering Metrics

Craig F. Merrill, Ph.D.

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# 1 EXECUTIVE SUMMARY

> It is sometimes felt that gerrymandering is such an obvious form of chicanery that any political body sincerely desiring to achieve a completely equitable redistricting should have no trouble in carrying out such a purpose . . . But whenever the drawing up of the boundaries is left even slightly to the discretion of an interested body, considerable latitude is left for the exercise of the art. [1]

The most obvious defense against partisan gerrymandering[1] is for disinterested parties to conduct redistricting processes in the open, using open source analytical tools, with the purpose of creating districts that will satisfy existing regulations and meet the intentions of both U.S. and state constitutions. Such a process would not eliminate the need for metrics capable of identifying partisan bias, but it would change how we use them. Rather than uncovering the unjust handiwork of partisan groups, the metrics would be used to guide the selection of the best redistricting products created by well-intentioned teams; a significantly easier threshold to meet (i.e., picking a good-enough redistricting map instead of trying to reveal the unknown intentions of a partisan group). Until the states institute such a process, we are left to rely on the courts to judge.

Analysts continue to develop and evaluate metrics that could be used to identify the existence of partisan gerrymandering. [2] However, one of the greatest difficulties they face is deciding how best to respond to the Supreme Court's desire for a simple measure to detect gerrymandered districts given the sophisticated tools being used to create them. The primary partisan gerrymandering metric used until recently has been symmetry; however, it has a number of weaknesses that keep it from serving as a legal standard. Namely, it relies on artificial data and, despite decades of use, has yet to identify a value above which partisan gerrymandering can be said to exist. Although it is fairly new, the efficiency gap metric has a significant body of research behind it. Moreover, it measures a quantity that is very similar to symmetry, only uses data from the election of interest, and it has a proposed standard. It is possible that the efficiency gap metric could fill this legal void. The challenge to Wisconsin's redistricting plan used the efficiency gap metric as the foundation of its argument for the existence of gerrymandering. Oral arguments were held in early October 2017, but it is unclear how the Court will rule.

Partisan gerrymandering metrics like those introduced here are the only means to ensure our democracy is based on "one person, one vote." Therefore, it is important for citizens to attempt to understand them.

[1] Gerrymandering is the process of manipulating the district boundaries of a state to favor or protect a specific group or individual. In the U.S., gerrymandering is most often exercised when redistricting Congressional, state legislature, or state judicial districts following the U.S. Census. Partisan gerrymandering generally seeks to establish districts that give an unfair advantage to a party (Democratic or Republican) or elected incumbents.

# 2 OBJECTIVE

This discussion seeks to provide a brief and accessible introduction to the metrics used to assess the presence of partisan gerrymandering in electoral redistricting. The paper includes a description of the electoral district features that are measured and the most common approaches used to measure them. It also presents a couple of newly developed techniques, one of which is now before the Supreme Court. We do our best to limit the amount of mathematics used to describe the metrics, and identify the critical strengths and weaknesses of each. The report also includes an appendix that provides a brief summary of the metrics.

For a nonpartisan introduction to the broader subjects of redistricting and gerrymandering, please visit the web-based information provided by the League of Women Voters.

# 3 BACKGROUND

The U.S. Constitution and most state constitutions impose constraints that require states to form electoral districts binding equal numbers of people, while using the simplest practical shapes. Additional regulations require that the districts be designed so all essential groups and people will have a say during elections.

The reason for imposing these constraints on the electoral redistricting process is to protect citizens against gerrymandering, which aims to establish unequal representation of the populace by manipulating district borders. Whether motivated by racism, political bias, or protection of incumbents, gerrymandering introduces geographic complexities, and these can be quantified in some fashion. However, the quantification effort is made a bit thornier by regulations (such as the Voting Rights Act) and additional desires (such as keeping municipalities or counties within a single district) that induce their own degree of boundary complexity.

This discussion is focused on the primary metrics used by analysts and the legal community to assess the presence of partisan gerrymandering. The methods measure the contiguity, compactness, and partisan makeup of state districts. The requirement for districts to be contiguous and compact help ensure their boundaries stay as plain as possible, and provide analysts with a “red flag” warning of possible gerrymandering. The partisan bias metrics measure the political affiliations and distribution of the people within the districts to spot concentrations or splits designed to favor a specific political party. In all cases, the only political parties large enough to be of concern in North Carolina are Democrat and Republican.

Complementing these metrics are modern computer approaches that can quickly generate thousands to millions of electoral districts using algorithms designed to optimize whatever characteristics the users specify. It is worth noting that it is these types of approaches (made possible by inexpensive leading-edge computer hardware and software) that made gerrymandering so pervasive following the 2010 census, and which continue to make it more difficult to detect the presence of gerrymandering using simple tools. Computers care not about the good or ill-intentioned motives of the programmers who create the algorithms that guide its processing, so they can be used to improve the redistricting process as well. We touch on this subject briefly at the end.

# 4 MEASURES OF GERRYMANDERING

## 4.1 CONTIGUITY

Calculating district contiguity is simple: either the district is a single region (pass) or it is composed of multiple separate regions (fail). However, there are occasions when it is not possible to create a contiguous district. Specifically, districts near bodies of water in which there are populated islands, or neighborhoods that are not connected (Figure 1). In these instances, it may not be possible for a district to be contiguous, and it may change how we apply some of the compactness metrics. Other than these types of cases, there are no good reasons why a district should be divided.

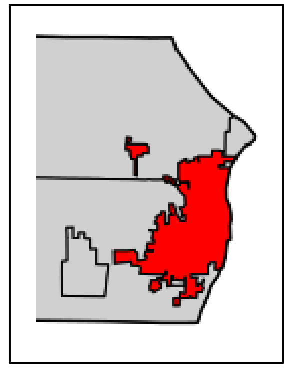

*Figure 1. Red region marks area of Racine, WI. [38].*

## 4.2 COMPACTNESS

A compact area has a specific definition in mathematics, but not so in politics. Generally speaking, the measure of an electoral district's compactness refers to the simplicity of its geometric shape. Figure 3 is an example of how a lack of simplicity can make gerrymandering visually apparent. However, to protect against more subtle forms of gerrymandering, researchers have developed methods to measure the compactness of a district by comparing its geometric characteristics to that of simpler shapes. For example, Reock developed a compactness metric that calculates the ratio of the area of the district to that of the smallest circle within which the district will fit. [2] [3] If the district is circular in shape, it's Reock ratio will be close to 1.0. Otherwise it will be less than 1.0 but greater than 0.0, with smaller values suggesting a greater likelihood of gerrymandering. In Figure 2 we show the Reock ratio geometry for North Carolina Congressional District 4 (115th Congress). [4] According to U.S. Census data, the district covers an area of 739 square miles and its greatest extent is 55 miles. To determine its Reock ratio, we set the diameter of the surrounding circle equal to this length and calculate the circle's area (2,376 square miles).[2] The result: 0.311. To get a sense of what this means, the Reock ratio for a square district is 0.637, and the average Reock ratio of all U.S Congressional districts up through 2013 is 0.405. [5] Because the ratio for District 4 is below the U.S. average, this should make one suspicious of gerrymandering.

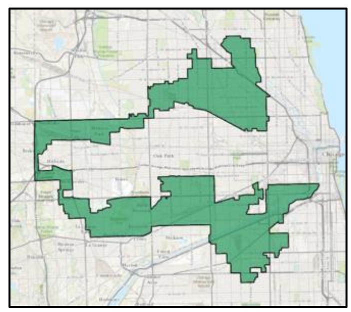

*Figure 3. Illinois' 4th Congressional district. An example of extreme gerrymandering that is readily identified via observation.*

Other compactness metrics evaluate the length of the perimeter of the boundary relative to simpler shapes. These can reveal gerrymandering effects missed by area comparisons, but they are also sensitive to how precisely the boundary is measured (you'll miss some of the squiggles if the boundary is measured at intervals of a tenth of a mile when compared to measurements performed every 10 feet). A few of these other approaches are discussed in the [appendix](#).

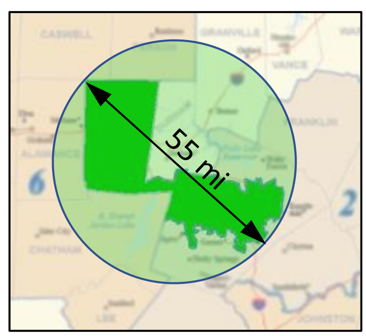


*Figure 2. Diagram of Reock ratio geometry for North Carolina's 2016 4th Congressional district.*

[2] The area of a circle, A, is $(\pi d^2)/4$, where d is the diameter.

A new compactness metric is the convexity coefficient. [6] [7] In mathematics, a convex shape is one in which all corners point outward (think triangle, square, hexagon). If we draw a line between any two points within a convex shape (Figure 4.a), the line itself will also be within the shape. However, if the district shape has indentations and protrusions, it is easy to see that a line between two points can exist outside of the shape, as shown in Figure 4.b. The convexity coefficient is a measure of the likelihood that a straight line between two points in a district will exit the district. The more complex the shape, the greater the number of boundary protrusions and indentations, which in turn increases the likelihood that lines drawn between points will exit the district. The coefficient can be modified so that it does not include shape complexities at state borders. In general, a district where more point-to-point lines can exit its area, is more likely to be gerrymandered. [6]

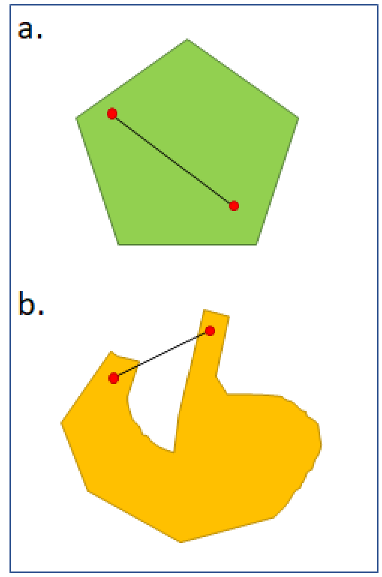


*Figure 4. A straight line between two points on an area. The line will remain within the area of a convex shape (a.), but can be outside of a shape that is not convex (b.).*

Slightly more complex compactness metrics measure the distribution of the population within districts. Gerrymandering often entails breaking up regions of concentrated populations, resulting in higher population concentrations on the district boundaries. If a district has larger population concentrations near its edges than in its central regions, there is a greater chance that it is gerrymandered.

Compactness metrics are necessary but not sufficient for identifying gerrymandering. To demonstrate this, let's consider a simple example: a square state with an evenly spread population (no areas of population concentration) but the population of the two parties ("Green" and "Yellow") are separated into nine equally populated regions (see Figure 5.a). [1] The state's total population is such that it has been assigned four congressional districts. Summing up the regions for both parties shows 5/9$^{\text{ths}}$ (≈56 %) of the population is Green and 4/9$^{\text{ths}}$ (≈44 %) Yellow. Given that there will be four districts, the fairest distribution would be to split the state such that two districts are Green and two are Yellow. But how should we execute the redistricting? Figure 5.b. shows one option: dividing the state in horizontal slices, will give two districts to each party. However, if we split the state vertically and horizontally as shown in Figure 5.c.—an approach that seems just as reasonable because it gives us nice compact square districts—the result is that each district has a higher Green population than Yellow, so we end up four Green districts and no Yellow districts. Moreover, applying compactness gerrymandering metrics to the boundaries of the districts in Figure 5.c. do not reveal any redistricting problems; the square state's districts are contiguous, compact, and have equal populations. Yet, it is apparent that this redistricting plan is a poor choice because 44% of the state's voters who happen to be Yellow party members will have no representation. Clearly, something more than a compactness measure is needed because they are blind to the extreme unfairness of this plan.

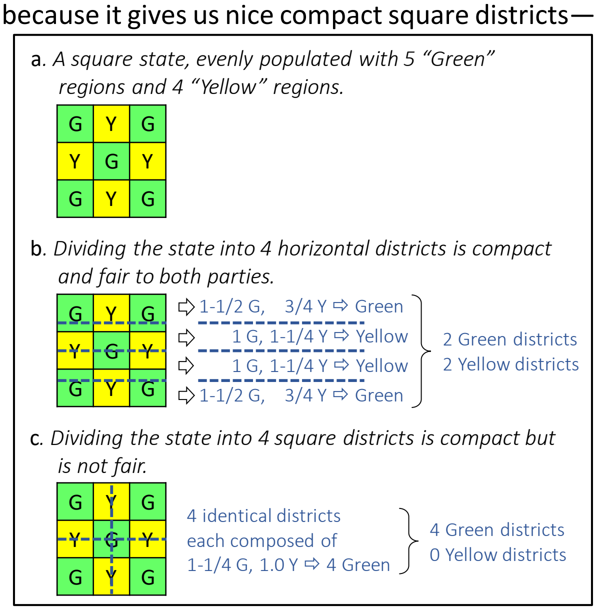


*Figure 5. Dividing a square state into 4 districts.*

Researchers recognize that all aspects of compactness cannot be described using a single metric, so multiple calculations using different compactness metrics will be needed to check for gerrymandering. Additionally, none of the compactness metrics have an agreed-to value below which a state's redistricting plan can be said to have an unacceptable level of partisan bias. Thus, researchers continue to explore means of understanding and interpreting compactness metrics. [8] To date, there are at least 20 different compactness measurement techniques under evaluation. [9] In the meantime, human judgement and multiple metrics will be needed to assess the compactness of redistricting plans.

## 4.3 Direct Measures of Partisanship

The objective of partisan gerrymandering is to make it easier for the party in control to win the maximum number of seats, resulting in disproportionate representation of the two parties. This can make the voice of the other party's voters unacceptably ineffective; a violation of the Constitution's Equal Protection Clause of the Fourteenth Amendment. [10] More recently, Justice Anthony Kennedy stated that the First Amendment "may be the more relevant constitutional provision in future cases that allege unconstitutional partisan gerrymandering." [11]

In 1986, the Supreme Court ruled that partisan gerrymandering is judiciable, and narrowly (5-4) maintained this position in 2004. However, the Court's ability to make a decision is thwarted because they lack a viable legal standard on which to pass judgement. [12] Until recently, the only partisan metric that has been used in court was symmetry. Symmetry measures how fairly the two parties are treated by comparing the average number of votes each required to win a seat. In an effort to address some of the shortcomings inherent in symmetry (discussed below), research focused on developing better methods for creating the data used in symmetry analysis, as well as forming new methods that attempt to directly measure the difference in voting efficiency between the two parties. [13] [14] [15]

There are currently four metrics being advocated for quantifying partisan bias: a statistically-based version of partisan symmetry, equal vote weight, the efficiency gap, and declination. We also provide an overview of responsiveness, which has an unclear role in assessing partisan gerrymandering but is often discussed alongside symmetry.

The partisan bias metrics include assumptions and simplifications that are important for us to understand. For example,

- They do not account for the strength or weakness of the candidate, campaign effectiveness, messaging effectiveness, scandals, or other current events that might influence how voters vote.
- They do not account for differences in voter turnout rates for different parties in different regions of a state; voter suppression; or, cross-over voting. [16] [17]
- They are designed to measure partisan bias in elections for two-party single member districts. If we look at the voter affiliations for North Carolina (Table 1), we see it is not a classic two-party system. There are only a small number of Libertarian Party voters, but unaffiliated voters are the second largest voting bloc in the state.

***Table 1. North Carolina voter registration as of Dec. 9, 2017.***

| Party | Number | Percent |
|---|---|---|
| Democrat | 2,646,757 | 39% |
| Republican | 2,064,050 | 30% |
| Libertarian | 34,223 | 1% |
| Unaffiliated | 2,087,142 | 31% |
| Total | 6,832,172 | |

### 4.3.1 Partisan Symmetry

Until recently, Constitutional authorities relied almost exclusively on the partisan symmetry metric to identify partisan gerrymandering. The symmetry metric shows how fairly the two parties are treated at the state level by comparing the average number of seats won with the average number of votes received. Figure 6 provides an example of the seats-to-votes curves that are used to determine the partisan symmetry value. The red curve in the figure represents a symmetric election since the number of votes obtained leads to the same number of seats won, regardless of the party. That is, if Green wins 47 % of the vote, they will receive 41 % of the seats; and if green wins 53 % of the vote, they win 59 % of the seats. Because the curve passes through the middle of the plot and is shaped the same on the top and the bottom, it is symmetric, which means Yellow will be treated the same as Green.

The purple dashed curve shows the relationship of an election with a partisan bias. For this case, if Green wins 50 % of the vote, they win 38 % of the seats; however, if Yellow wins 50 % of the vote, they win 62 % of the seats—a strong partisan bias in favor of Yellow. It is important to note that the Supreme Court has stated that the seats won in a state do not have to be proportional to the votes received, but the ratio of the seats won to the votes received must the same for both parties. [12]

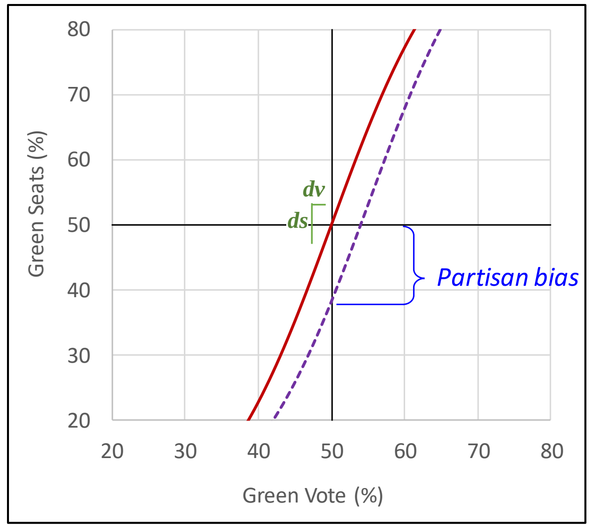

*Figure 6.Some of the partisan metrics can be described by way of a plot of a seats vs. votes curve for one of the parties. Here we show **symmetry** which estimates partisan bias, and **responsiveness**, the slope of the seats-votes curve. The solid red curve is a symmetric (nonpartisan) response. The dashed purple curve contains a partisan bias in favor of the Yellow.*

A statewide election only produces a single point for the seats to votes plot shown in Figure 6. To create the curve used for the partisan symmetry calculation, analysts generate additional data points using information from similar but different elections. The use of this made-up data has been a troubling issue for the Supreme Court. In 2006, a 5-4 plurality of the Court held the position that partisan gerrymandering was judiciable; but, simultaneously stated that symmetry alone was not sufficient because: 1) it relies on "conjecture" about the relationship between seats won and votes cast; and, 2) it has not yet provided a judiciable standard for determining partisan gerrymandering. [18] Many researchers contend that it is appropriate to generate data points as is done for the symmetry metric, but it is not evident that the Court can be persuaded. [2]

### 4.3.2 Responsiveness (Swing Ratio)

Responsiveness—sometimes called swing ratio— is a measure of the competitiveness of an election. Referring again to Figure 6, responsiveness is the change in seats won relative to the change in the votes received; it is the rate of change of the number of seats a party wins as a function of the votes they receive ($ds/dv$). The responsiveness of most U.S. elections is two to three, meaning that a one percent change in vote leads to a two to three percent change in the number of seats won. [19]

The responsiveness metric is often presented along with the symmetry metric because it uses the same seats-to-votes curve; however, it is a distinctly different measurement. [20] It is generally viewed as being most useful for gauging gerrymandering designed to protect incumbents, because this type of gerrymandering acts to make it more difficult for the vote to swing in the direction of the challenger. [21]

## State Election Data

Table 2 provides data that will be used to help describe three partisan gerrymandering metrics: equal vote weight, efficiency gap, and declination. The table contains election results for an imaginary state consisting of ten legislative districts. There are two parties in competition for seats; a "Green" party and a "Yellow" party. The rows containing Green data are marked with green circles and those of the Yellow party with yellow squares. We will use these symbols in a later plot as well.

The top two rows of the table contain the results of the election where the party that won the district are highlighted in light cyan. The lower left corner of the table summarizes the results of the race: Green won three of the ten races (30 percent of the vote) and received 50.7 percent of the vote. The results of partisan gerrymandering metrics that will be discussed in the following sections are highlighted in light purple box in the lower right corner of the table.

The vote numbers are small to keep the table neat. However, multiplying them by 100,000 makes the returns representative of what one might see in a modern U.S. election.

*Table 2. Imaginary state election data.*

| DISTRICT | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | TOTAL |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Green votes | **279** | 172 | 167 | 148 | **185** | 139 | 169 | 179 | **234** | 178 | 1,850 |
| Yellow votes | 120 | **198** | **192** | **212** | 180 | **193** | **201** | **206** | 99 | **199** | 1,800 |
| Total votes | 399 | 370 | 359 | 360 | 365 | 332 | 370 | 385 | 333 | 377 | 3,650 |
| Min votes to win | 200 | 185 | 180 | 180 | 183 | 166 | 185 | 193 | 167 | 189 | 1,828 |
| Green vote % | 69.9% | 46.5% | 46.5% | 41.1% | 50.7% | 41.9% | 45.7% | 46.5% | 70.3% | 47.2% | 50.7% |
| Green wasted votes | 79 | 172 | 167 | 148 | 2 | 139 | 169 | 179 | 67 | 178 | 1,300 |
| Yellow wasted votes | 120 | 13 | 12 | 32 | 180 | 27 | 16 | 13 | 99 | 10 | 522 |
| Green waste % | 19.8% | 46.5% | 46.5% | 41.1% | 0.5% | 41.9% | 45.7% | 46.5% | 20.1% | 47.2% | 35.6% |
| Yellow waste % | 30.1% | 3.5% | 3.3% | 8.9% | 49.3% | 8.1% | 4.3% | 3.4% | 29.7% | 2.7% | 14.3% |
| District Efficiency Gap | 10.3% | -43.0% | -43.2% | -32.2% | 48.8% | -33.7% | -41.4% | -43.1% | 9.6% | -44.6% | -16.3% |

| | |
|---|---|
| Green won | 3 of 10 |
| Green % won | 30.0% |
| Green vote % | 50.7% |

| | |
|---|---|
| Green district median % | 46.5% |
| Green district mean % | 50.6% |

| | |
|---|---|
| Efficiency Gap | -16.3% |
| Equal Vote Wt | -4.1% |
| Declination | 0.70 |

### 4.3.3 Equal Vote Weight

Unlike symmetry, the equal vote weight metric focuses on the votes (not seats) by measuring the difference between a party's statewide median (the middle value) and mean (the average value) vote percentages. [22] As an example, consider the state election data in Table 2 where the bottom center of the table shows the median and mean values for the Green party, with the equal vote weight result of -4.1 %. The negative value means the district configuration favors the Yellow party.

The creators of the equal vote weight metric have proposed three factors to determine if gerrymandering is present, and two additional ones for deciding if the level is high enough to support a legal finding of gerrymandering. Therefore, this metric addresses two important shortcomings in the partisan symmetry metric: it does not rely on data generated using the results from other races, and it has a possible means for establishing a judiciable standard. The primary weakness of the equal vote weight metric is that it may require numerous election cycles to reveal partisan bias. The developers of the method call it a "leading indicator" of possible gerrymandering. [23] It has not been used in court.

### 4.3.4 Declination

A new partisan gerrymandering metric appeared in the literature this year. This approach uses the vote fractions for one of the parties to calculate the declination; an angle that roughly describes how the vote percent changes between the districts that were lost and those that were won by a party (Figure 7). [24] We would not expect to see different angles for the districts lost (below 50 percent) and won (above 50 percent) in an ungerrymandered state. [24] Therefore, if the value for declination is not close to zero, then gerrymandering may be present.

Declination is easy to calculate, makes no assumptions about voter affiliation, and does not rely on data generated from multiple elections. It does not yet have an agreed-to standard, but the developers do provide a version of declination, $\tilde{\delta}$, that makes it possible to compare values with those of other states.

Based on an evaluation of nearly 1,200 elections, the authors suggest that values of $\tilde{\delta}$ greater than 0.47, should be investigated for gerrymandering. [24]

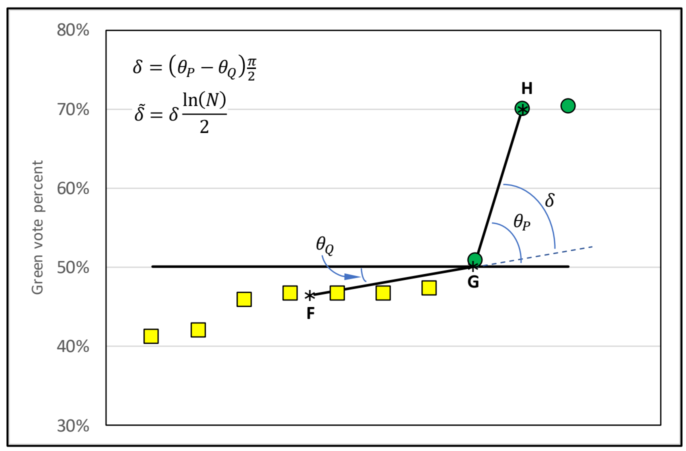


*Figure 7. Declination uses the Green vote percent from the two-party election data in Table 2. The yellow squares are districts won by Yellow and the Green circles are those won by Green. For this data $\delta$ = 0.61 and $\tilde{\delta} = 0.70$. Based on reference [24], this districting falls well above the threshold for partisan gerrymandering in favor of the Yellow.*

### 4.3.5 Efficiency Gap

Partisan gerrymandering is an effort to make the votes more efficient for the party in control. The gerrymanderers seek to win the majority of the statewide elections with the smallest practical vote margins; and, cause the other party to win fewer races but by relatively large vote margins. Thus, the party in power will "waste" fewer votes in victory <u>and</u> in defeat. With this perspective in mind—and inspired by the Court's openness to adopting a gerrymandering standard in *LULAC v Perry* (2006)—researchers developed the efficiency gap, a metric intended to quantify the difference in voting effectiveness between the two parties. [14] [15] As such, it measures a quantity very similar to symmetry, but it does so using a more direct approach and without the use of data derived from different elections.

The efficiency gap calculation is straightforward. Consider the two-party election shown in Table 2. We see that Green won 1,850 votes (50.7 %) but only three seats. On the other hand, Yellow won 1,800 votes (49.3 %), but was able to secure seven seats. After summing up the wasted votes for each party, the overall efficiency gap for the election is calculated to be 16.3 % in favor of Yellow. This is at least 8 % higher than the level the metric's developers think warrants additional scrutiny for a state of this size. [25]

In October 2017, the Supreme Court heard oral arguments in *Gill v Whitford*, the Wisconsin redistricting lawsuit that uses efficiency gap as the partisan gerrymandering metric. From the oral argument, it was apparent that the Court was concerned that the metric is new and that the derivation of the recommended standard (the point at which one would say gerrymandering is excessive) is too complicated to be manageable by the Court. [26] There was also concern that in unusual situations (districts with highly non-competitive races or many closely split districts), the efficiency gap result was not be consistent. However, based on assessments from various researchers, it seems evident that the efficiency gap will be a useful tool for assessing partisan gerrymandering, but its use will need to include "further statistical testing and modeling" to ensure the results are proper. [27]

Returning to Table 2 for a moment, it is evident that the imaginary state election data is extremely partisan. The vote was evenly split, but the seat distribution was not; and, all three metrics give a strong indication of a partisan bias in favor of Yellow.

## 4.4 Computational Approaches

The gerrymandering that occurred following the 2010 census exploited new computational methods that were not available for use during earlier post-census redistricting planning. Many of these methods used "big data" analytical techniques that enable multiple computers and graphical processing units to distribute the workload, and to apply machine learning algorithms to reveal relationships between data elements that would otherwise go unnoticed. These tools gave gerrymanderers a huge advantage because it enabled them to use previously inaccessible highly detailed data (geographic information system data, market surveys from industry, census data, and state voter registration records) to form districts to their clients' liking. In some instances, it also made the gerrymandering more difficult to detect. However, these same tools can be applied to detect gerrymandering, identify optimal redistricting plans, and even generate districts that best support constitutional objectives in ways that would not occur to analysts. [28]

Computational approaches provide a more powerful means of applying and interpreting metrics used to assess gerrymandering. What is more, they can enable redistricting teams to produce many potential redistricting plans, and to create distributions of metric values, making it possible to identify minimally acceptable values for each metric.

There are various numerical approaches and although the computations are complex, the computer programming is not. If the redistricting process uses open source software, the programs it implements can easily be inspected and understood by others with experience in the given programming language. Mexico has been using computers for redistricting for over 20 years. [29] Duke University's Center for Political Leadership, Innovation, and Service, as part of their "Beyond Gerrymandering" project, used computers to generate thousands of possible districts for North Carolina and the results selected compared favorably with those created by a nonpartisan team of retired judges. [30]

# 5 Discussion

The most obvious defense against partisan gerrymandering is for disinterested parties to conduct redistricting processes in the open, using open source analytical tools, with the purpose of creating districts that will satisfy existing regulations and meet the intentions of both U.S. and state constitutions. Such a process would not eliminate the need for metrics capable of identifying partisan bias, but it would change how we use them. Rather than uncovering the unjust handiwork of partisan groups, the metrics would be used to guide the selection of the best redistricting products created by well-intentioned teams; a significantly easier threshold to meet (i.e., picking a good-enough redistricting map instead of trying to reveal the unknown intentions of a partisan group). Until the states institute such a process, we are left to rely on partisan gerrymandering metrics.

Analysts continue to develop and evaluate metrics that could be used to identify the existence of partisan gerrymandering. However, one of the greatest difficulties they face is deciding how best to respond to the Supreme Court's desire for a simple measure to detect gerrymandered districts given the sophisticated tools being used to create them. Metrics designed to measure contiguity, compactness, symmetry, and responsiveness come from an era when complex calculations were not possible. Of these, only symmetry is designed to measure partisan bias, but it has a number of vulnerabilities that keep it from serving the Court as an acceptable standard. Namely, it relies on artificially generated data and, despite decades of use, has yet to identify a manageable standard. More recent metrics such as equal voter weight, declination, and efficiency gap address these two shortcomings. The voter weight metric is only considered to be a red flag of potential gerrymandering, and the declination metric just appeared this year. Although it is also fairly new, the efficiency gap metric has a significant body of research behind it; and, unlike voter weight and declination, the quantity measured by efficiency gap is very similar to symmetry.

Narrow pluralities of the Supreme Court have contended that partisan gerrymandering is judiciable, but also concede they are unable to make a definitive ruling because symmetry lacks a manageable standard. Previous rulings have looked favorably on the symmetry metric, but its dependence on sources of data that are not actually from the election being analyzed, and the lack of a standard have prevented the Court from using it in a decision. It is possible that the efficiency gap metric could fill this legal void. The efficiency gap measures a quantity that is very similar to symmetry, but it does not require made-up data, and it has a proposed standard.

The challenge to Wisconsin's redistricting plan used the efficiency gap metric as the foundation of its argument for the existence of gerrymandering. Oral arguments were held in early October 2017, and it is unclear how the Court will rule. However, the interest has led to surge in research and analysis of the metric. [21] [23] [27] [29] [31] [32] [33] [34] [35] [36] [37]

It is unfortunate that the tools needed to prevent partisan gerrymandering are fairly complex and can be difficult for people to understand. However, metrics like those discussed here are the only means to ensure our democracy is based on "one person, one vote." Therefore, it is important for citizens to make an attempt to understand them.

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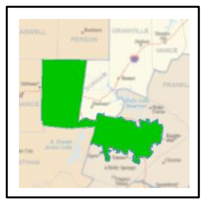
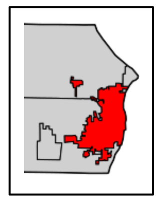
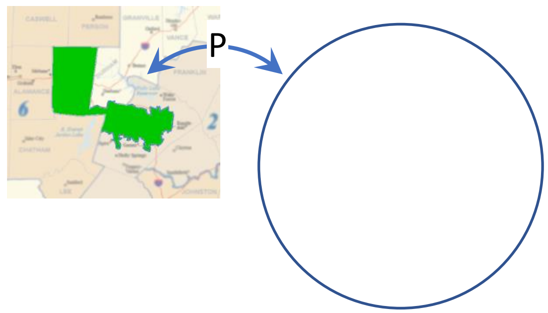
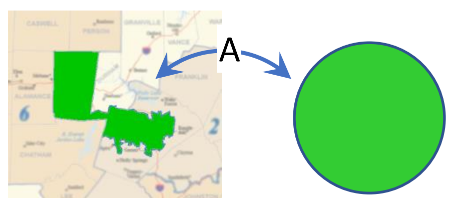
# 7 APPENDIX – PARTISAN GERRYMANDERING METRICS

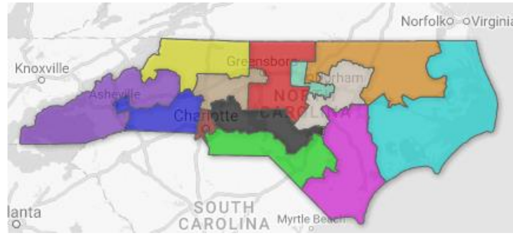
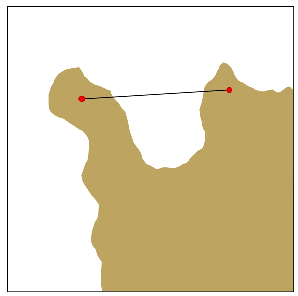
The following table gives a brief overview of the most widely used or discussed metrics for revealing gerrymandering of legislative districts. Contiguity and compactness are used to identify most types of gerrymandering, but are used in conjunction with partisan gerrymandering metrics when assessing state districts for partisan bias.

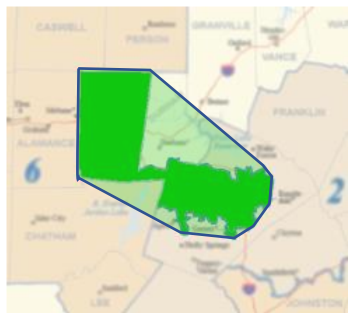
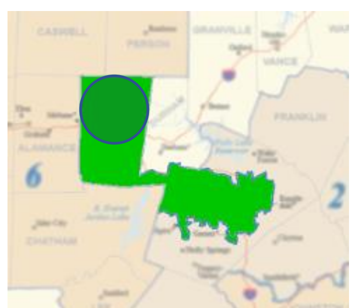
The quoted statement at the beginning of each type of measure comes from the North Carolina General Assembly's "Overview of the Redistricting Process" (https://www.ncleg.net/representation/Content/Overview.aspx). The statements are the regulatory and/or guiding reason the metrics are necessary, but they do require the metrics be used.

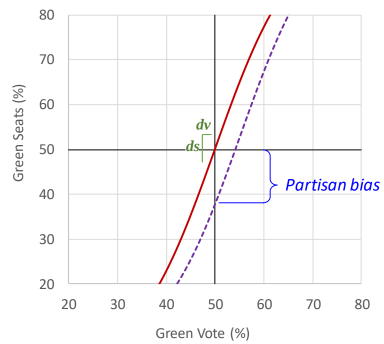
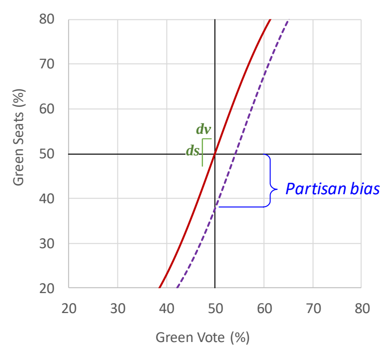
For accessible explanations of the metrics by others see Azaveo (2009) and Austin (2014).

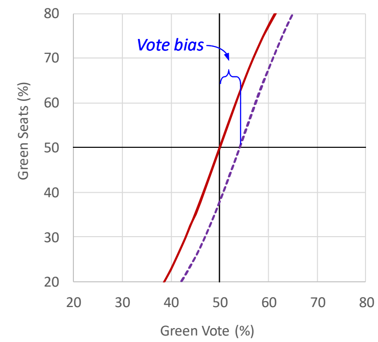
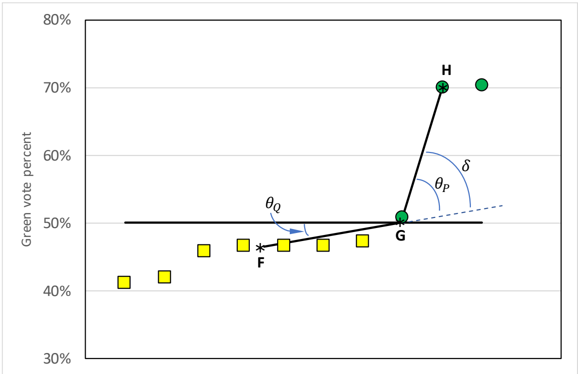
| Name | Description | Diagram | Equations | Discussion |
|---|---|---|---|---|
| *Contiguity – "Under the State Constitution, Senate and House districts must consist of contiguous territory. By tradition, the contiguity requirement also has been applied to Congressional districts. Contiguity means that all parts of a district must touch. The district must not have any detached parts."* | | | | |
| Contiguity | Ensure there are no unavoidable separate or detached parts of the district. | Contiguous  Discontiguous  | None | This is a visual test, but one must use care to zoom in on areas were boundaries come very close to one another. Sometime contiguity cannot be met, such as in the case of states with islands or separated towns. |

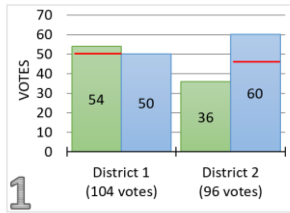
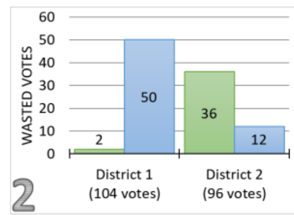
| Name | Description | Diagram | Equations | Discussion |
|---|---|---|---|---|
| *Compactness – "First, the General Assembly should draw the districts required by the Voting Rights Act. Second, it should take all the counties with just the right population to be single-member districts and make them one-county single-member districts. Third, it should take all the counties that have just the right populations for one or more districts and divide those counties into compact single-member districts. Fourth, for the remaining counties it should group them into clusters of counties and divide the clusters into compact single-member districts, crossing county lines within the cluster as little as possible."* Gerrymandering grows the boundary (perimeter) of a district much more than the area. Compactness metrics attempt to capture this effect. The ratio scores are between zero and one, with scores closer to one being simpler, less gerrymandered, shapes. | | | | |
| Polsby-Popper ratio | The ratio of the area of a circle to that of a district with the same perimeter, P. This is a measure of the amount of contortion (zig-zag) of district boundaries. |   | $P_{district} = P_{circle}$ <br> $\frac{4\pi A_{district}}{P^2}$ <br> Ratio varies from 1.0 for a circle and approaches 0.0 as shape becomes less compact. | Small perimeter variations (i.e., less smooth perimeter) will cause large reductions in the ratio. Natural boundaries (i.e., coastlines) can substantially reduce the ratio as well. A lower ratio implies a greater likelihood of gerrymandering. <br> Results are sensitive to how precisely boundary is measured with greater precision leading to lower ratios. |
| Schwartzberg ratio | The ratio of the perimeter, P, of the district to that of a circle with the same area, A. This is a measure of the amount of contortion (zig-zag) of district boundaries. |   | $A_{district} = A_{circle}$ <br> $\frac{\sqrt{4\pi A}}{P}$ <br> Ratio varies from 1.0 for a circle and approaches 0.0 as shape becomes less compact. | Similar to Polsby-Popper except that it is more sensitive to perimeter variations. <br> Results are sensitive to how precisely boundary is measured with greater precision leading to lower ratios. |

| Name | Description | Diagram | Equations | Discussion |
|---|---|---|---|---|
| Perimeter test | The sum of the perimeters of all districts in the state. This is a measure of the amount of contortion (zig-zag) of district boundaries. |  | $P = \sum_{d} P_d$ | The more variations in the district perimeters, the greater this value will be. It is useful for comparing different redistricting efforts for a state, but cannot be broadly interpreted or compared to other states' districts.<br><br>Results are sensitive to how precisely boundary is measured with greater precision leading to lower ratios. |
| Convexity coefficient | A measure of the likelihood that a straight line between two points in a shape will exit the shape. The more complex the shape, the greater the likelihood that a line exists outside of the district boundary. |  | $\frac{1}{A_D}\iint A_D(x,y)dxdy$<br><br>Value varies from 0 for a convex shape and approaches 1.0 as shape becomes more compact. | Able to capture small perimeter variations, with values closer to one suggesting gerrymandering is more likely. This method can remove the contribution of state and natural boundaries,<br><br>It is more difficult to calculate and more difficult for the layman to interpret the results. |

| Name | Description | Diagram | Equations | Discussion |
|---|---|---|---|---|
| Reock ratio | The ratio of the district area, A (or population, $\mathcal{P}$) to that of the smallest circle that completely encloses the district. This is a measure of the dispersion of the area (population) of the district. |  | $\frac{A_{\text{district}}}{A_{\text{circumscribed}}}$ <br> $\frac{\mathcal{P}_{\text{district}}}{\mathcal{P}_{\text{circumscribed}}}$ <br> Ratio varies from 1.0 for a circle and approaches 0.0 as shape becomes less compact. | Reock tends is most sensitive to districts that are long and narrow, or discontiguous, both of which lead to lower ratio values. To minimize impact of the latter, separate areas (islands) are drawn with a separate circle and the areas (populations) added together. |
| Convex Hull | The ratio of the district area, A (or population, $\mathcal{P}$) to that of a convex polygon that encloses the district. This is a measure of the dispersion of the area (population) of the district. |  | $\frac{A_{\text{district}}}{A_{\text{polygon}}}$ <br> $\frac{\mathcal{P}_{\text{district}}}{\mathcal{P}_{\text{polygon}}}$ <br> Ratio varies from 1.0 for a convex shape and approaches 0.0 as shape becomes less compact. | Imagine pulling a rubber band around the district. This method is similar to Reock, but less sensitive to long narrow shapes. Islands can be handled separately (as for Reock). |
| Ehrenburg ratio | The ratio of the area (population) of the largest circle that can be inscribed in the district to that of the district area (population). This is a measure of the dispersion of the area of the district. |  | $\frac{A_{\text{inscribed}}}{A_{\text{district}}}$ <br> $\frac{\mathcal{P}_{\text{inscribed}}}{\mathcal{P}_{\text{district}}}$ <br> Ratio varies from 1.0 for a circle and approaches 0.0 as shape becomes less compact. | This ratio more clearly captures districts with tentacles because only one of the tentacles will contain the largest inscribed circle. Lower values imply more tentacles and the more likely the presence of gerrymandering. |

| Name | Description | Diagram | Equations | Discussion |
|---|---|---|---|---|
| *Partisan Metrics – "As they redraw districts, legislators will be urged by various people and groups to consider additional redistricting principles: 'Keep communities in the same district.' 'Retain the cores of incumbents' prior districts.' 'Increase or reduce the strength of one or the other political parties.' Redistricting is a complex and political process, and all of these motives are legitimate and traditional redistricting considerations. Ultimately, the voters, through their elected representatives, control the process."* This statement makes clear that partisan gerrymandering is permissible, but may not be desirable. | | | | |
| Symmetry | A curve fitted to the seats vs. votes points (where the points are created using made-up data). Shows how fairly the two parties are treated by comparing the seat share relative to the percent of the vote received. |  | $$\frac{S}{1\text{-}S}=\alpha\left(\frac{V}{1\text{-}V}\right)^{\beta}$$ $\alpha$=1 implies no partisan bias (e.g., the solid red curve).<br><br>Purple dashed-curve shows negative bias; i.e., biased in favor of the other party. | The metric emphasized since the Court ruled partisan gerrymandering judiciable in 1986. The solid red curve represents perfect fairness for both parties. This is desirable but not usually achievable—there is always some amount of partisan bias (dashed purple line), but there is no standard as to how much is acceptable. It also requires the use of made-up data. |
| Responsiveness (Swing ratio) | The rate of change of a party's seats as a function of votes; i.e., the slope of the symmetry curve at a point on the curve. |  | $$\beta=\frac{ds}{dv}$$ In the U.S., $\beta$ is typically between 2 and 3 seats/vote percentage. | This is a measure of how competitive a state is; that is, how well the seats won respond to the party voters.<br><br>This is most often used to reveal gerrymandering intended to protect incumbents.<br><br>It requires the use of made-up data. |

| Name | Description | Diagram | Equations | Discussion |
|---|---|---|---|---|
| Equal vote weight | Focuses on the votes by finding the difference between a party's statewide median and mean vote percentages. |  | $\text{Median}_{\text{district}} - \text{Mean}_{\text{district}}$ | The mean value represents the strength of one partisan group of voters; the median is what it takes to win half the seats. Therefore, the greater the difference in these two values implies all votes are not being weighted the same and suggests a greater likelihood of gerrymandering.<br>This is a fairly new metric and the developers see it as a "red flag" for gerrymandering. It also does not use made-up data. |
| Declination | Measures asymmetry in the vote distribution by examining the slope of a party's vote fraction when the districts are arranged in order from the lowest to the highest fraction. The change is slow will be zero for the perfect case without gerrymandering. |  | $\delta = (\theta_P - \theta_Q)\frac{\pi}{2}$<br>$\tilde{\delta} = \delta\frac{\ln(N)}{2}$ | If $\lvert\tilde{\delta}\rvert \gtrsim 0.47$, the redistricting should be investigated for potential gerrymandering.<br>Declination is very easy to calculate, and does not use made-up data. However, it is extremely new and has not undergone critical review. |

| Name | Description | Diagram | Equations | Discussion |
|---|---|---|---|---|
| Efficiency gap | Measures the fraction of votes that are wasted by each party, where wasted votes are those that are 1) cast for a losing candidate; or 2) cast in excess of what is needed to win. | Diagram for a two-district state. The wasted votes shown in Figure 2 show that the Blue party has many more (62 vs. 38) wasted votes, suggesting the election is biased in favor of Green.     | $W_d = \frac{V_{lost} + V_{excess}}{V_{total}}$<br>$E = \sum_d W_d$<br>$EG = E_{Rep} - E_{Dem}$<br>Higher EG implies gerrymandering more likely, but maximum value will vary by state size. | Gerrymandering seeks to make the other party's vote inefficient. Therefore, unbiased states will have approximately the same efficiency for both parties, leaving the gap close to zero.<br>Efficiency gap is the metric being used in *Gill v Whitford*, the Wisconsin redistricting lawsuit.<br>Efficiency gap is easy to calculate, but must be used carefully in extreme cases. The characteristic measured is similar to symmetry, but it does not use made-up data and has a proposed standard. |